Island formation in disordered superconducting thin films at finite magnetic fields

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It has been predicted theoretically and observed experimentally that disorder leads to spatial fluctuations in the superconducting (SC) gap. Areas where SC correlations are finite, coined SC islands, were shown experimentally to persist into the insulating side of the superconductor-insulator transition. The existence of such (possibly weakly coupled) SC islands in amorphous thin films of superconducting material accounts for numerous experimental findings related to superconductor-insulator transition and non-monotonic magneto-resistance behavior in the insulating region. In this work, a detailed analysis pertaining to the occurrence of SC islands in disordered two-dimensional superconductors is presented. Using a locally self-consistent numerical solution of the Bogoliubov-de-Gennes equations, the formation of SC islands is demonstrated, and their evolution with an applied perpendicular magnetic field is studied in some detail, along with the disorder-induced vortex-pinning. While mean-field theory cannot, in principle, explore phase correlations between different islands, it is demonstrated that by inspecting the effect of a parallel magnetic field, one can show that the islands are indeed uncorrelated SC domains. Experimental predictions based on this analysis are presented.

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I. INTRODUCTION

Interplay between disorder and superconductivity has been at the focus of attention for quite a long time. More than four decades ago, it has been established that the effect of weak disorder on superconductivity is not substantial.[1,2] Strong disorder, however, may have a profound effect, driving the system from a superconducting (SC) to an insulating state. Such a superconductor-insulator transition (SIT) was observed in two-dimensional amorphous SC films.[3] Upon decreasing the film thickness or increasing a perpendicular magnetic field, these films (which are held below their bulk critical temperature) exhibit a transition from a SC state, characterized by a vanishing resistance as $T \to T_c$, to an insulating state, in which the resistance diverges as $T \to 0$. The possibility of tuning the system continuously between these two extreme phases may be a manifestation of a quantum phase transition (which, strictly speaking, occurs at zero temperature).[4]

Despite the substantial amount of experimental data, and numerous theoretical investigations, there are still several unresolved issues pertaining to the physics of these systems. One of the main puzzles is the mechanism by which the magnetic field destroys the SC correlations.

The "dirty boson" theory,[5,6] describing bosons (Cooper pairs) in a random potential, regards the SIT as a transition into a Bose-glass phase, in which the pairing amplitude is finite in the sample but its phase is strongly fluctuating, giving rise to an insulating state. Such a phase is characterized by a pair-vortex duality, and its hallmark is a universal value for the resistance $R_Q = h/4e^2$ at the transition. While some experiments are consistent with that prediction,[7–9] others are not.[10,11] Beside the issue of universal resistance, there are other observations which cannot be simply explained by the dirty boson model, for instance, the temperature dependence of the crossing point and the classical XY critical exponent.[12]

The role of the large fluctuations in the local SC gap, Δ, and the formation of SC islands have been emphasized in several other works.[13–16] In Ref. 13 the authors predict the existence of such islands by calculating the mesoscopic fluctuations of the order-parameter at the mean-field level. Since phase fluctuations are ignored by mean-field calculations, it was pointed out that there is no real SC transition within that approximation. Later it has been shown[14] that inclusion of quantum fluctuations (beyond mean field) indeed results in a quantum phase-transition. A detailed mean-field study of SC correlations in disordered two-dimensional films has been presented in Ref. 15, employing a locally self-consistent solution of the mean-field Bogoliubov-de Gennes (BdG) equations[17] at finite temperature and at zero magnetic field. It was indeed found that disorder induces strong fluctuations of Δ in space, resulting in regions with large

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order parameter (here interpreted as SC islands - SCIs), separated by regions of vanishingly small order parameter. The role of temperature is found[16] to be similar to that of disorder.

Recently we have incorporated thermal phase fluctuations beyond the BdG theory,[18] and have demonstrated the formation of isolated SCIs within the two-dimensional disordered negative-U Hubbard model: increasing magnetic field leads to the loss of SC phase correlations between different areas in the sample and an eventual SIT. The island picture is very helpful in explaining some of the experimental puzzles. The coexistence and competition between Cooper pair and electron (or quasi-particle) transport give rise to the non-universality of the critical resistance. It can also explain[19] the non-monotonic magneto-resistance behavior observed on the insulating side of the SIT.[20] The theory also predicts the persistence of the SC islands into the insulating phase. Such SC correlations in the insulating phase have been observed experimentally by using AC transport (and measuring the superfluid stiffness on the insulating phase)[21] and by Aharonov-Bohm interference (where oscillations with period $h/2e$ in the magnetoresistance were observed in the magneto-resistance on the insulating phase, indicating the presence of Cooper pairs).[22]

In this picture, the approach to the SIT from the insulating side is governed by the emergence of phase correlations between the islands as temperature or magnetic field are lowered, eventually resulting in a macroscopically coherent sample. This picture is in agreement with the (quantum) percolation scenario.[23,24] Evidence for percolation-like behavior is substantiated in several experiments.[8,9,11,25,26] Interestingly, the theory also demonstrated that phase correlations between different islands may be explored *even within mean-field theory*, by looking at the response to parallel (Zeeman) magnetic field (see below). Thus, in principle, the SIT can also be investigated by solving the BdG equation in a finite magnetic field. In this work, however, we focus on the effects of magnetic field on the formation and evolution of SC islands in highly disordered two-dimensional SC systems in a wide range of temperatures. (We separate the effects of magnetic field into orbital field and Zeeman field, which we sometimes refer to as perpendicular and parallel fields, even though each of the latter leads to both orbital and Zeeman effects.)

The structure of this paper and the main achievements are listed below. In section II the model and method of calculations are briefly described. In section III A the role of perpendicular magnetic fields is studied (assumed to act on the orbital degrees of freedom as mentioned above). In particular, it is found that, as the field is increased, the size of the SC islands shrinks and the strength of the local order parameter diminishes. It is explicitly shown that vortices are formed and their disorder-induced pinning is analyzed. Moreover, these vortices tend to accumulate in places where, in the absence of magnetic field, the order parameter was low (that

is in the "normal regions" of the sample). The presence of these vortices suppresses the Josephson couplings between the SCIs, leading eventually to completely decoupled islands. Section III B is devoted to examining the role of a parallel magnetic field, manifested by a Zeeman splitting. It is found that at low temperatures and weak orbital magnetic fields, weakly disordered systems exhibit an abrupt vanishing of the SC order parameter in response to increasing parallel field, resembling a first order transition, as expected for a uniform system.[27] On the other hand, when either temperature, disorder, or the orbital field are increased, the system undergoes a series of transitions, as a function of parallel field, corresponding to the sequential vanishing of Δ on separate islands, in agreement with Ref. 18. The emergence of local magnetic order in areas where superconductivity is destroyed is demonstrated. In Section IV we summarize and discuss the results.

II. MODEL AND BASIC EQUATIONS

In this section, the model and method of calculations are briefly introduced. The starting point is the effective tight-binding BdG Hamiltonian[17] on a square lattice,

$$\mathcal{H}_{\text{BdG}} = - \sum_{<ij>,\sigma} t_{ij}(c_{i\sigma}^{\dagger}c_{j\sigma} + \text{H.c.}) + \sum_i (\epsilon_i - \tilde{\mu}_i)\hat{n}_{i\sigma}$$
$$+ \sum_i [\Delta(\mathbf{r}_i)c_{i\uparrow}^{\dagger}c_{i\downarrow}^{\dagger} + \Delta^*(\mathbf{r}_i)c_{i\uparrow}c_{i\downarrow}] \quad , \text{(1)}$$

where the sum $<ij>$ runs on nearest neighbors, ϵ_i are random site energies taken from a uniform distribution $[-W, W]$, with W being the disorder strength, $\tilde{\mu}_i = \mu + \frac{|U|}{2}\langle n_i \rangle$ is the local Hartree-shifted chemical potential,

$$\Delta(\mathbf{r}_i) = -|U|\langle c_{i\downarrow}c_{i\uparrow} \rangle \quad \text{(2)}$$

is the local pairing potential and $U < 0$ is an effective electron-electron on-site attractive interaction.[17] The orbital magnetic field enters via the nearest-neighbor hopping integral t_{ij}. In the Landau gauge it can be written within the Peierls substitution rule as

$$t_{ij} = t_0 \exp(-i\alpha y_i) \quad \text{(3)}$$

if $\mathbf{r}_i - \mathbf{r}_j$ is in the **x**-direction, and $t_{ij} = t_0$ otherwise. In Eq. (3) $\alpha = \phi/\phi_0$ where ϕ is the magnetic flux per plaquette, $\phi_0 = hc/e$ is the flux quantum, and y_i is the **y**-direction coordinate (in units of lattice spacing). Hereafter, t_0 is set to unity and serves as reference to all other energy scales. Using a Bogoliubov transformation one can diagonalize $\mathcal{H}_{\text{BdG}}$ exactly and obtain the BdG equations,

$$\begin{pmatrix} \hat{\xi} & \hat{\Delta} \\ \hat{\Delta}^* & -\hat{\xi}^* \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix} \quad . \quad \text{(4)}$$

In Eq. (4), $\hat{\xi}u_n(\mathbf{r}_i) = -\sum_{\hat{\delta}} t_{i,i+\delta}u_n(\mathbf{r}_i + \hat{\delta}) + (\epsilon_i - \tilde{\mu}_i)u_n(\mathbf{r}_i)$ where $\hat{\delta} = \pm\hat{\mathbf{x}}, \pm\hat{\mathbf{y}}$, and $\hat{\Delta}u_n(\mathbf{r}_i) = \Delta(\mathbf{r}_i)u_n(\mathbf{r}_i)$

and similarly for $v_n(\mathbf{r}_i)$, and the energies are the quasiparticle (QP) excitation energies $E_n \geq 0$. The pairing potential $\Delta(\mathbf{r}_i)$ and the electron density per site n_i are given, in terms of the QP amplitudes $u(\mathbf{r}_i)$ and $v(\mathbf{r}_i)$,

$$\Delta(\mathbf{r}_i) = |U(\mathbf{r}_i)| \sum_n u_n(\mathbf{r}_i) v_n^*(\mathbf{r}_i)$$

$$\langle n_i \rangle = 2 \sum_n |v_n(\mathbf{r}_i)|^2 \quad . \tag{5}$$

In what follows, Eq. (4) is solved numerically on a finite (albeit large) two-dimensional square lattice with hard-wall boundary conditions, starting with some initial guesses for $\Delta(\mathbf{r}_i)$ and n_i. $\Delta(\mathbf{r}_i)$ and n_i are then determined via Eq. (5), and the procedure is repeated until self-consistency is obtained *on each site*. In order to be able to compare the same system at different magnetic fields, the chemical potential is altered in every iteration such that the total average density per site $\langle n \rangle = \frac{1}{N} \sum_i n_i$ remains constant (N being the total number of sites in the lattice).

III. NUMERICAL RESULTS

A. Effect of orbital magnetic field

We start by examining the effect of an orbital magnetic field on the pairing potential Δ. In Ref. 19 it was argued that the SCIs diminish in size and may even vanish as the magnetic field is increased. This assumption is indeed confirmed by the numerical results. Figure 1 depicts the spatial distribution of the order parameter amplitude, $|\Delta(\mathbf{r})|$, in the two-dimensional system, for three different values of the magnetic field. At zero magnetic field the order parameter fluctuates in space due to disorder, assuming larger values (bright areas in Fig. 1) in places where the effective local potential is small, i.e., $\epsilon_i - \tilde{\mu}_i \approx 0$,[15,28] which facilitates fluctuations between 0 and 2 electrons on the site. These areas are separated by regions with small order parameter (dark areas in Fig. 1) and hence constitute "superconducting islands" (as will be shown in following sections). As the magnetic field is increased, the size of these islands and the strength of the order parameter diminishes, and some SCIs virtually disappear (that is, at very strong fields the order parameter there becomes smaller than the numerical tolerance). It should be pointed out that while the average value of the order parameter diminishes by nearly 50% when the flux is increased from $\phi/\phi_0 = 0$ to $\phi/\phi_0 = 0.2$, the maximal value of the order parameter drops by less than 10%, meaning that the concept of "SC islands" persist well within the strong field region (it is the loss of correlations between the islands that drives the system into its normal state).

The distribution function of the order parameter amplitude, $P(|\Delta|)$, is plotted in Fig. 2 for different values of
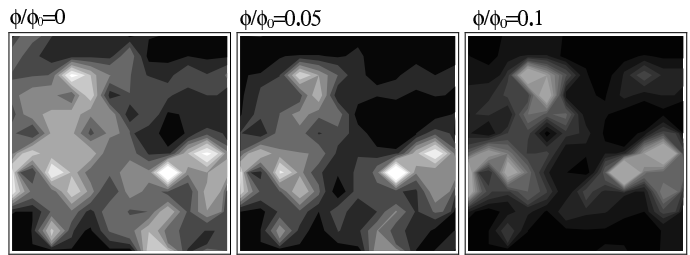

FIG. 1: Spatial distribution of the order parameter amplitude $|\Delta|$ for different values of the orbital magnetic field, $\phi/\phi_0 = 0, 0.05, 0.1$. Bright regions, corresponding to large values of $|\Delta|$, constitute "superconducting islands", separated by regions of small $|\Delta|$ (dark regions). Increasing magnetic field leads to attenuation of the SC order parameter and even to the vanishing of SC order in some areas of the sample. Calculation is performed on a 12×12 size lattice with electron density $\langle n \rangle = 0.875$, disorder strength $W = 1$ and interaction strength $U = 2$ (this value for U is maintained throughout).

the magnetic field. As can be seen, the distribution function shifts from being peaked around some finite initial value to being peaked at $\Delta = 0$. However, the distribution tails persist even at high-field, indicating that SC islands are present.
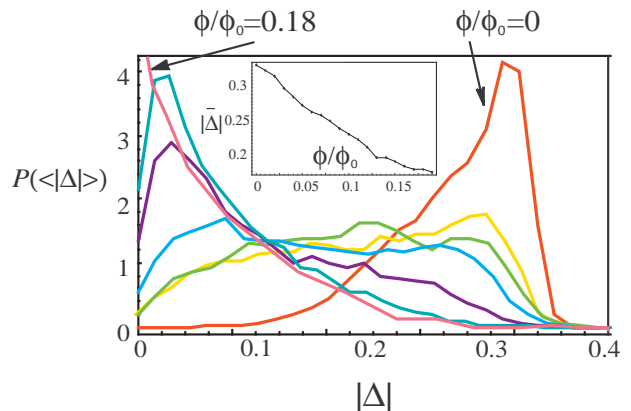

FIG. 2: (Color online) Distribution function of the SC order parameter amplitude $|\Delta|$, for seven different values of magnetic field, $\phi/\phi_0 = 0 - 0.18$ (calculated for a system size 16×16, averaged over the entire lattice and over 25 realizations of disorder). With increasing magnetic field the distribution shifts from being peaked at finite value (the uniform-system value) to being peaked at zero value. Note, however, the persistence of tails at large $|\Delta|$ even at high magnetic field, indicating the existence of regions with large $|\Delta|$. Inset: the average order parameter amplitude, $\langle|\Delta|\rangle$, as a function of ϕ/ϕ_0, showing a decrease in the SC order parameter with the applied orbital magnetic field.

In the inset of Fig. 2 the average order parameter amplitude $\langle|\Delta|\rangle$ is plotted as a function of ϕ/ϕ_0. We note that (i) in a finite lattice $\langle|\Delta|\rangle$ never strictly vanishes and (ii) for a given disorder realization, $\langle|\Delta|\rangle$ may be a non-monotonic function of ϕ/ϕ_0. In fact, in a clean system $\langle|\Delta|\rangle$ has been shown to oscillate as a function of

magnetic field.[29] Disorder, however, tends to smear the non-monotonicity, and upon averaging over disorder realizations one obtains a monotonically decreasing function.

Another illustration of the interplay between disorder and magnetic field is manifested in the location of the vortices. In a clean system one expects the vortices to form a regular Abrikosov lattice. However, strong disorder modifies the position of vortices, as they are pinned within regions where the amplitude of the order parameter has a small value. In Figure 3 the phase of the SC order parameter is plotted for different values of the magnetic field corresponding to the consecutive insertion of one additional flux quantum into the system. As can be seen, a new vortex enters the system each time an additional flux quanta is inserted, and vortices do not form a lattice. Rather, their position is affected by disorder.
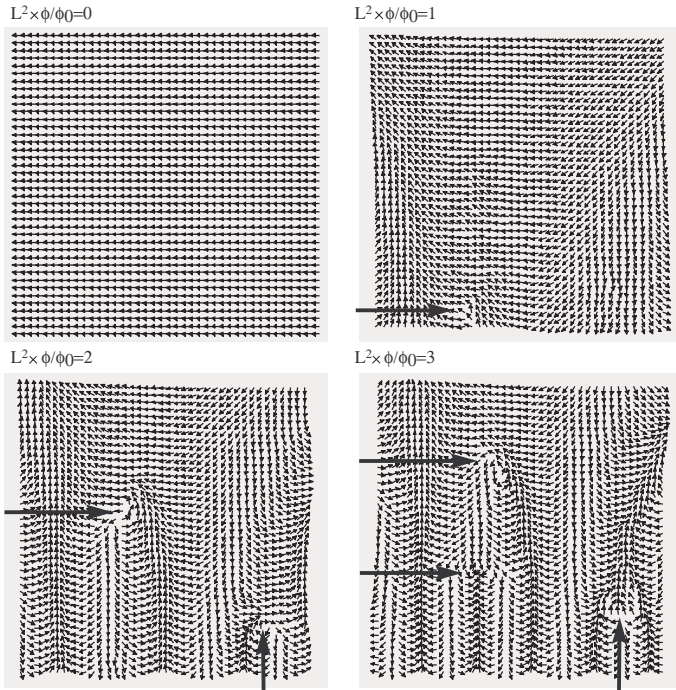

FIG. 3: Spatial mapping of the SC order parameter phase for values of magnetic field corresponding to a different number of flux quanta in the system (i.e., $\phi/\phi_0 \times (L \times L) = 0, 1, 2, 3$), on a 40×40 size sample. Arrows point at the positions of the vortices. For each number of flux quanta, a similar number of vortices is formed, their position is irregular and determined by the disorder potential.

These correlations can be exposed more clearly. In Figure 4 the order parameter amplitude at zero field is plotted (Fig. 4(a)), together with the phase of the order parameter (Fig. 4(b)), for the same system as in Figure 3 with two flux quanta. As can be seen, the vortex cores are positioned where the amplitude is low (dark regions in (a)), i.e., they are "pinned" to regions with low order parameter. The emerging interactions between vortices still play a significant role, as, for example, the second

vortex that enters the system changes the position of the first one.

In Ref. 18 it was shown that the SIT is driven by loss of phase-coherence between the SC phases on different islands. The fact that vortices appear in regions of low order parameter amplitude, i.e. in between the SC islands, is consistent with the picture that orbital magnetic field causes loss of phase coherence between the islands: the effective Josephson coupling between the islands is reduced with increasing magnetic field, until the Josephson energy becomes smaller than temperature (or the amount of quantum phase fluctuations) and the islands decouple.
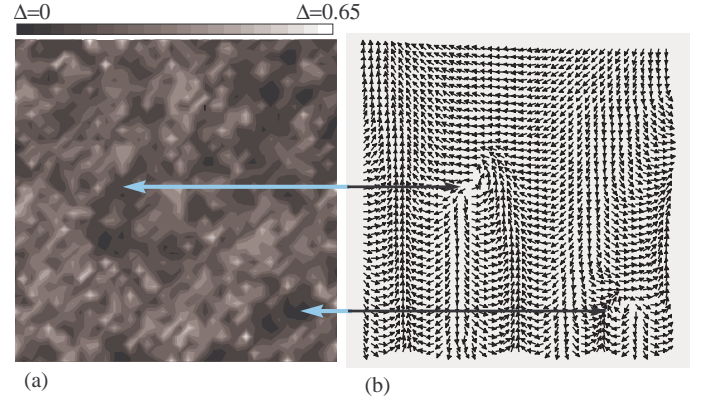

FIG. 4: Comparison between (a) the order parameter amplitude at zero field and (b) the phase of the order parameter at finite field (two flux quanta), indicating that the vortices are formed at the position of weak superconducting order (dark regions in (a)).

B. Evolution of superconducting islands in a parallel magnetic field

In the previous subsection the roles of disorder and orbital magnetic field in generating fluctuations in the amplitude of the SC order parameter have been demonstrated. However, the question remains whether these fluctuations may be regarded as separated SC islands. Here we demonstrate how the Zeeman effect resulting from an application of a parallel magnetic field helps to answer this question. This is carried out by extending our previous calculations,[18] and include the electron spin through the Zeeman effect.

When a parallel magnetic field is applied, the single electron levels split. It was shown a long time ago[27] that upon applying a parallel field, a uniform superconductor undergoes a first-order transition. The reason for this transition is simple: superconductivity requires occupation of paired spin up and spin down states, which becomes costly with increasing Zeeman energy. For a single pair, one expects the transition to take place when the gain from superconductivity, Δ_0, (the bulk value of the order parameter), is overcome by Zeeman energy,

$h_c \equiv g\mu_B B_{||}$, where $B_{||}$ is the strength of the parallel field, g being the g-factor for electrons and μ_B is the Bohr magneton. In fact, when correlation effects are taken into account, one finds that the transition from superconductivity to a magnetically aligned state occurs at $h_c = \Delta_0/\sqrt{2}$. For ultra-small isolated SC grains it was shown[30] that a finite size effect (manifested in a level spacing of the order of Δ_0) may smooth the otherwise sharp transition.

In order to account for the effect of parallel field, a Zeeman splitting term $\sum_{i\sigma} \sigma h c_{i\sigma}^\dagger c_{i\sigma}$ is added to the Hamiltonian of Eq. (1). The Bogoliubov transformation now has to be performed with spin-dependent quasi-particle amplitudes $u_{n\pm}(\mathbf{r}_i)$ and $v_{n\pm}(\mathbf{r}_i)$, which now obey the spin-generalized BdG equations,[31]

$$\begin{pmatrix} \hat{\xi} \pm h & \hat{\Delta} \\ \hat{\Delta}^* & -\hat{\xi}^* \pm h \end{pmatrix} \begin{pmatrix} u_{n\pm}(\mathbf{r}_i) \\ v_{n\pm}(\mathbf{r}_i) \end{pmatrix} = (E_n \pm h) \begin{pmatrix} u_{n\pm}(\mathbf{r}_i) \\ v_{n\pm}(\mathbf{r}_i) \end{pmatrix} . \quad (6)$$

Once these equations are solved, the local pairing potential and occupation are determined by self-consistent equations similar to that of Eq. (5). Note that for singlet (S-wave) pairing, the different spin-generalized quasi-particle amplitudes are decoupled, and the coupling between the different spins enters only in the determination of the self-consistent Hartree shift and order parameter.

For a clean or weakly disordered system and for parallel field strength $h < h_c$, the eigenvectors are independent of h (due to the simple dependence of the energies on the Zeeman term), and hence the solution for the pairing amplitude $\Delta(\mathbf{r})$ and the local occupation is unchanged. On the other hand, for $h > h_c$ a solution with $\Delta(\mathbf{r}) = 0$ yields a lower free energy. Thus, one should observe a step-like behavior of the order parameter. In contrast, in a disordered system, as $\Delta(\mathbf{r})$ varies in space, one also expects h_c to be space-dependent. For a system composed of separated SCIs, one thus expects that the order parameter on each island will vanish when $h > h_c(i)$ (where i is an index for identifying the corresponding island). Thus, the vanishing of the average value of Δ is expected to exhibit multiple steps, each step corresponds to the vanishing of superconductivity in a different island.

In Figure 5 the average order parameter amplitude $\langle|\Delta|\rangle$ is plotted as a function of h for different orbital fields. One can clearly see that for small ϕ (weak perpendicular magnetic field), the system undergoes a single sharp transition. In contrast, for finite values of ϕ, step structure of $\langle|\Delta|\rangle$ is indeed visible. This is seen more clearly in the inset of Figure 5 where $\langle|\Delta|\rangle$ is plotted as a function of h at a perpendicular field $\phi/\phi_0 = 0.018$, with much higher resolution in h. At this resolution, the step-like behavior is seen even for this high value of the orbital field. Thus, large orbital field suppresses SC correlations between the islands. They seem to lose SC order separately and successively, depending on the specific value of the gap for each SCI.

To make this visual, the spatial distribution of the pairing parameter at $\phi/\phi_0 = 0.014$ is plotted in Figure 6 for
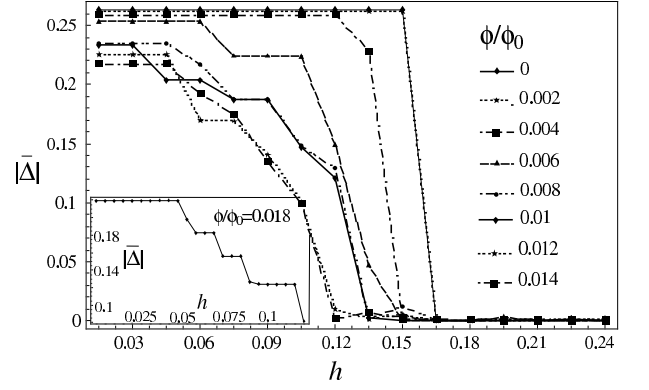
FIG. 5: The average order parameter amplitude $\langle|\Delta|\rangle$ as a function of the Zeeman splitting h for different values of orbital magnetic field, $\phi/\phi_0 = 0 - 0.014$. For small ϕ/ϕ_0, the system undergoes a first-order transition, which is smoothed as ϕ/ϕ_0 increases. For $\phi/\phi_0 = 0.004$ steps in $\langle|\Delta|\rangle$ are visible. Inset: the same quantity for $\phi/\phi_0 = 0.018$, but with larger resolution in h. At this resolution, steps in the $\langle|\Delta|\rangle$ are clearly seen.

$h = 0 - 0.012$ (Fig. 6(a-h)). The consecutive vanishing of the order parameter in different areas with increasing parallel magnetic field is clearly visible. It is noticeable that even when the order parameter vanishes, as a result of the field, in one area of the sample, the other regions remain almost unaffected. The different regions (for which the order parameter vanishes at different parallel fields) can thus be indeed regarded as separated SCIs.
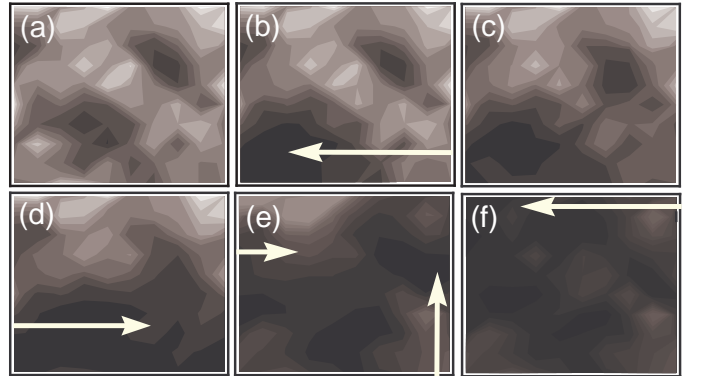
FIG. 6: Spatial distribution of the order parameter amplitude $|\Delta|$ at $\phi/\phi_0 = 0.014$ for $h = 0 - 0.12$ (a-f). Arrows indicate regions in which the order parameter vanishes abruptly (SC islands).

As already mentioned, another indication for the destruction of the SC order may be elucidated by examining the local magnetization $M(\mathbf{r}) = \langle n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})\rangle$, which develops locally where SCIs are destroyed by the parallel field. This is shown in Figure 7, where we plot side by side the spatial distributions of the *differential variations* of the magnetization and of the pairing potential, for a finite orbital field and for different Zeeman fields (same
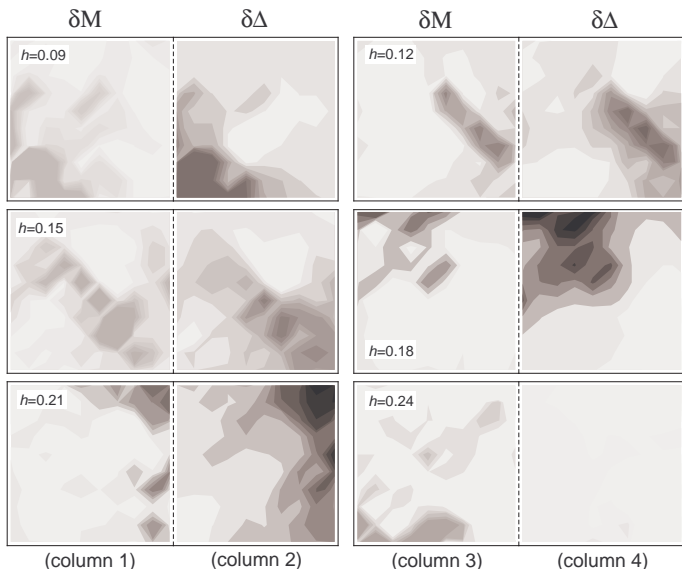
FIG. 7: Spatial distribution of the differential changes in magnetization, $\delta M(\mathbf{r}, h) \equiv M(\mathbf{r}, h) - M(\mathbf{r}, h - 0.03)$ (columns 1 and 3), and in pairing potential amplitude, $\delta(\mathbf{r}, h) \equiv \Delta(\mathbf{r}, h) - \Delta(\mathbf{r}, h - 0.03)$ (columns 2 and 4), for different Zeeman fields $h = 0.045 - 0.12$, and for orbital field $\phi/\phi_0 = 0.014$. The emergence of magnetic correlation occurs simultaneously and is highly correlated with the vanishing of SC order.

values as in Fig. 6). It is clearly seen that the magnetization (dark regions in the left panels) develops only where SCIs (dark regions in the right panels) shrink. Notice that even when the parallel field strength exceeds the critical value ($h_c \approx 0.24$ for these parameters), the magnetization still fluctuates, now due to correlations with the local electron density.

An important and experimentally testable prediction of the above calculation is that a strongly disordered SC film subject to a parallel field may have both a zero resistance state and finite magnetization. This is in contrast with a clean film, which is either SC or spin polarized.

IV. SUMMARY AND DISCUSSION

In this paper the physics related to formation of SCIs in disordered two-dimensional SC films and their evolution under an applied magnetic field was addressed. SCIs are defined as specific domains exhibiting large value of the order parameter amplitude surrounded by other domains for which the amplitude of the order parameter is small. Using the locally self-consistent solution of the BdG equations, it was demonstrated that SCIs are formed in disordered SC films. Upon an increase of the orbital magnetic field, the size of these islands tend to decrease, together with the strength of the order param-

eter. This is accompanied by loss of SC phase correlations between different islands.

In order to verify that SCIs indeed constitute separate isolated domains, the application of parallel magnetic fields has been included in our numerical simulations. For a uniform or weakly disordered sample it results in a first order transition into a normal state. In contrast, for medium and high disordered systems and in the presence of an orbital magnetic field, it is found that upon increasing the parallel magnetic field, the average order parameter vanishes in a series of steps, indicating that each island undergoes a phase transition at its own turn, with its own critical parallel field. This substantiates the picture of well separated and isolated SCIs. In Ref. 18 we have demonstrated a one-to-one correspondence between the phase correlations between the islands, at finite orbital field and zero Zeeman field, on one hand, and their sequential response to the Zeeman magnetic field, on the other hand. This suggests that the SIT can be studied within mean-field theory, even though the latter does not include phase fluctuations.

The above results can in principal be addressed experimentally, by means of local measurements using, e.g., scanning tunneling microscopy, which is also sensitive to magnetic order. We predict that, as a function of parallel magnetic field, the (average) SC order parameter will exhibit a behavior resembling that of Figure 5, that is the transition will be smoothed when the orbital effects are dominant. The local SC gap on an isolated island, on the other hand, should exhibit a sharp attenuation characteristic of a first-order transition. This will be accompanied by local formation of magnetic order, so we expect local magnetism coexisting with either bulk superconductivity (if the SCIs percolate) or local superconductivity (if they do not).

Lastly we point out that some of the phenomena discussed here were also observed in high-T_c superconductors. Local variations in the SC gap have been observed by STM measurements.[32] In the underdoped region, it was observed[33] that local SC gap persists well into the normal phase. Even more relevant is the observation[34] in these materials of inhomogeneous magnetic response, which persists above T_c. Thus disorder may play an important role in high-T_c superconductors, especially in the underdoped regime, leading, for example, to the interpretation of the pseudo-gap, as an average over the spectra of SC and non-SC regions.

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